FOUNDATION STUDIES IN FUGUE

by

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ABBREVIATIONS

WTC I  Well-Tempered Clavier, Volume I
WTC II  Well-Tempered Clavier, Volume II

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FOREWORD

This book on one of the most important aspects of music composition has a two-fold purpose. First, it is meant to provide the student with a working technique in fugue so that he can approach the composing of a fugue systematically and with a well-defined order of steps.

Secondly, it is intended that through the observations presented, the student can formulate a method for examining fruitfully the fugues of the great composers of both the past and present.

Two pieces of advice may be in order. The first is that before undertaking the study of fugue, the student should be well equipped in all aspects of harmony and counterpoint, and especially canon. A deficiency in any of these techniques will make itself evident in fugal composition which calls for the ultimate in smoothness and elegance. Secondly, the student must not feel that this book is the last word in the study of fugue. On the contrary, it is only the beginning. There are so many excellent fugues to be studied and so many other treatises to be read, but—as the title implies—it is hoped that the present book, small as it is, will provide a solid foundation upon which to build.

HUGO NORDEN

Boston, Massachusetts
April 1977
CHAPTER I
The Subject

The theme upon which and around which a fugue is constructed is commonly identified as the SUBJECT. A melody to be suitable for this purpose will generally exhibit three characteristics:
1. it is usually brief, but embodies a complete musical idea;
2. it is easily recognizable as it weaves through the contrapuntal texture, and
3. it lends itself well to contrapuntal and harmonic manipulation and exploitation.

From the craftsman's point of view the last attribute is the most important. Bach, as also did other composers, employed subjects that range in length from a few notes

Ex. 1  
WTC I, Fugue No. 9 in E major

to several measures.

Ex. 2  
WTC II, Fugue No. 24 in B minor

Rhythmically, a subject may be of the utmost simplicity,

Ex. 3  
WTC I, Fugue No. 4 in C-sharp minor

or of considerable complexity.

Ex. 4  
WTC II, Fugue No. 17 in A-flat major

In the matter of tonality, a subject may be starkly diatonic,

Ex. 5  
WTC II, Fugue No. 9 in E major

or highly chromatic.

Ex. 6  
WTC I, Fugue No. 14 in F-sharp minor

Ex. 7  
WTC II, Fugue No. 6 in D minor
It may even be a "tone row," employing all twelve notes.

Ex. 8  WTC I, Fugue No. 24 in B minor

A subject may flow continuously as in those quoted above, or it may be fragmented as in the following.

Ex. 9  WTC I, Fugue No. 22 in B-flat minor

Ex. 10  WTC II, Fugue No. 20 in A minor

Ex. 11  WTC II, Fugue No. 11 in F major

A useful feature of a successful subject is a strong opening motive that can be readily identified by the listener so that he will know when the entrance is being made. This is sometimes referred to as the 'head' of the subject. French treatises speak of this device as "La tête du sujet." The following are typical.

Ex. 12  WTC I, Fugue No. 5 in D major

Ex. 13  WTC I, Fugue No. 3 in C-sharp major

Ex. 14  WTC II, Fugue No. 7 in E-flat major

Ex. 15  WTC II, Fugue No. 5 in D major

Ex. 16  WTC I, Fugue No. 16 in G minor

The foregoing general observations are introductory. It is now necessary to examine in detail the specific types of subjects and their respective answers.

CHAPTER II

The Answer

The Answer, under the simplest conditions, is the subject transposed to the dominant key; that is, a perfect 5th higher or a perfect 4th lower.

Ex. 17  WTC II, Fugue No. 9 in E major

When a subject in a major key contains the leading-tone, this will require an accidental for the leading-tone of the dominant key.

Ex. 18  WTC I, Fugue No. 9 in E major

When a subject is in a minor key, accidentals will be required for both the II and VII notes of the dominant key.

Ex. 19  WTC I, Fugue No. 4 in C-sharp minor

When the subject contains additional accidentals, that is, any besides those enumerated above, these too must be reflected in the dominant key. The two following subjects and their respective answers illustrate this principle.

Ex. 20  WTC I, Fugue No. 14 in F-sharp minor

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Ex. 19  WTC I, Fugue No. 4 in C-sharp minor

When the subject contains additional accidentals, that is, any besides those enumerated above, these too must be reflected in the dominant key. The two following subjects and their respective answers illustrate this principle.

Ex. 20  WTC I, Fugue No. 14 in F-sharp minor
In the above instance, Bach ends the subject on the III note of D minor and the answer on the I note of A minor. An answer that is a literal transposition into the dominant key is identified as Real. When, under specific conditions that are described in detail below, certain notes in the subject that are in the dominant key and answered in the tonic key in contrast to the rest of the subject, the resulting answer is said to be Tonal. The following examples illustrate how this principle of alternating tonality operates.

**Subjects That Require A Real Answer**

Subjects in this group will meet two general conditions:
1. they will begin and end on the I or III notes, and
2. they will not contain a leap from the I or III notes to the V or VII notes at or near the beginning.

(a) Begins on I and ends on III

(b) Begins on I and ends on III

(c) Begins on III and ends on I

(d) Begins on III and ends on III

The entire system of subjects that require real and tonal answers is crystallized into 18 specific types in the illustrations that follow.
Subjects That Require A Tonal Answer

Group I

Subjects of this kind will begin with one or more notes in the dominant key and complete the subject in the tonic key. Whatever is in the dominant key will be answered in the tonic key, and vice versa.

(a) Begins on V and ends on I

Ex. 29  WTC I, Fugue No. 3 in C-sharp major

The following subject of this same type is especially interesting because of the chromatic flow of the melody.

Ex. 30  WTC I, Fugue No. 12 in F minor

(b) Begins on V and ends on III

Ex. 31  WTC II, Fugue No. 2 in C minor

In contrast to the preceding illustrations which have only one dominant note at the beginning that serves merely as an anacrusis, the following two subjects feature a much longer opening section in the dominant key. Both quotations are from fugues by Bach.

Ex. 32  Organ Fugue in E minor

The next two subject types are so rare as to be virtually non-existent in practical usage by the masters. The examples provided below are synthetic models contrived to illustrate the theoretical principle involved.

(c) Begins on VII and ends on I

Ex. 34  Norden

(d) Begins on VII and ends on III

Ex. 35  Norden

When Bach employed subjects that begin with VII, he considered the leading-tone merely as an anacrusis in the tonic key, and not as the III of the dominant key. The following two subjects by Bach are given interesting and unexpected answers: the first being given a Real answer, while the second is answered in the subdominant key.

Ex. 36  WTC II, Fugue No. 13 in F-sharp major
Subject-VII of Subdominant Key

Group II

Subjects in this group begin in the tonic key and end in the dominant key, while their answers do exactly the opposite, beginning in the dominant key and ending in the tonic key.

(a) Begins on I and ends on V

Ex. 38

A spectacular subject and answer of this type appears in Fugue No. 11 in F major of Klengel’s Kanons und Fugen, Volume I.

Ex. 39

In Variation 26 of the Goldberg Variations Bach employs the following subject in which the dominant key is not introduced until the third note from the end. It is quite unusual for the dominant key to be introduced so late.

Ex. 40

(b) Begins on I and ends on VII

Ex. 41

The following is a longer and more involved subject of the same type.

Ex. 42 Klengel: Kanons und Fugen, Vol. I, Fugue No. 17 in A-flat major

Subjects that begin on III and end on either V or VII are very rare. The examples that follow are again synthetic models contrived to demonstrate the principle involved.

(c) Begins on III and ends on V

Ex. 43

(d) Begins on III and ends on VII

Ex. 44

Ambiguous tonality comes about in a minor key when a subject of type (d) above does not contain the leading-tone of either the tonic or dominant key. To illustrate, the following subject begins on the III of D minor and ends on the III of A minor, the natural leading-tone of D minor. However, it would be entirely reasonable to view the subject as beginning on I of F major and ending on the I of C major, being the same as V of F major.
But, when essentially the same problem contains the raised leading-tones of the minor keys involved, no such ambiguity exists.

Ex. 46

Norden

Group III

Group III of subjects that require a tonal answer combines the properties of Groups I and II. A subject in Group III consists of three sections:

1. An opening effect in the dominant key as in Group I,
2. A middle section in the tonic key, and
3. A closing section in the dominant key as in Group II.

The following illustrations are self-explanatory.

(a) Begins on V and ends on V

Ex. 47

WTC I, Fugue No. 7 in E-flat major

(b) Begins on V and ends on VII

Ex. 50

(c) Begins on VII and ends on V

Ex. 51

(d) Begins on VII and ends on VII

Ex. 52

Unlike (a) of this group, (b), (c) and (d) are rarely, if ever, encountered in fugues by the masters. The above three examples are provided in order to demonstrate the theoretical possibilities that subjects of this kind afford. The composer should feel free to use any kind of subject that meets his artistic requirements, whether or not it is to be found in the fugal literature.
Group IV
(a) Any of the preceding sixteen types—four Real and twelve Tonal—
containing a leap from the I or III notes to the V or VII notes at or near
the beginning. The following are typical.

Ex. 53  WTC I, Fugue No. 8 in E-flat minor

Except for the second note, V approached by upward leap from I, this subject
is in effect a Real (a). Thus, the second note is a dominant “island,” as it
were, within a subject that is otherwise in the tonic key.

Ex. 54  WTC I, Fugue No. 22 in B-flat minor

Here the second note is a V approached by a descending leap from I within
a subject that begins on I and ends on III as in Real (b).

While the two preceding subjects featured only one V note, the following
has a full measure of V, approached by upward leap from the initial I
anacrusis effect. In the two preceding examples, the V note was unaccented
in relation to the preceding I.

Ex. 55  Bach, Organ Fugue in C minor

Instead of only one V note being approached by leap from the opening I,
the following example contains a four-note motive in the dominant key and
which is approached by descending semitone from the initial I to the VII.
(cf. the first example of Group II(a.).

Ex. 56  WTC I, Fugue No. 23 in B major

The following answer is most unusual. The subject is Tonal, Group I (b),
beginning on V and ending on III. The third note is V, preceded by VI. The
unusual feature here is that this V is answered tonally in spite of the tradi-
tional rule that a V or VII after II or VI is to be answered Real and
not tonally.

Ex. 57  WTC I, Fugue No. 21 in B-flat major

In Fugue No. 2 of WTC I Bach makes an interesting and very deliberate
distinction between the V being approached by leap from I in contrast to it
being approached by leap from II. The two quite different situations are
pointed out in the following illustration.

Ex. 58  WTC I, Fugue No. 2 in C minor

Numerous instances are to be found in which a V or VII preceded by
I or III is not answered tonally. The subject of the Art of the Fugue opens
with the ascending leap from I to V, and is answered in the academically
correct manner with the V answered by I as in the above examples;

Ex. 59

but when a rhythmic variant of the same subject appears in Contrapunctus
11 this leap from I to V is ignored and it is treated with a Real answer.

Ex. 60
(b) The entire subject answered in its subdominant key. Subjects answered in this way usually begin on V or VII. The following examples show how this answering process comes about.

Ex. 61

Bach, Partita IV

Ex. 62

Bach, Violin Fugue in G minor

This subject and answer is used in the same way in the Organ Fugue in D minor. See also the last example given under Group I.

In Fugue No. 13 in F-sharp major of Klengel's Kanons und Fugen, Vol. II, the following unusual subject-answer relationship occurs. The subject is constructed according to the specifications of Group I (b), beginning on V and ending on III. Ordinarily only the first note would be answered a 4th higher, but in this case the entire subject is answered a 4th higher so that the answer terminates on the III note of the subdominant key of B major.

Ex. 63

Modulating Effects To The Dominant Key

Every subject that ends on V or VII—that is, the I or III of the dominant key, requires a modulation from the tonic key. Five thematic devices serve this purpose. These are demonstrated in the following abstract and brief examples.

1. In an ascending major 2nd from accented to unaccented notes.

Ex. 64

See Ex. 42, 43, and 52.

2. Between repeated notes from accented to unaccented.

Ex. 65

See Ex. 44.

3. In a conspicuous leap, usually (but not necessarily) from unaccented to accented notes.

Ex. 66

See Ex. 39, 41, 45, 49, and 50.

4. After a tied or dotted note or rest, provided it does not damage a suspension.

Ex. 67

See Ex. 47.

5. In a descending minor 2nd, often from accented to unaccented notes.

Ex. 68

See Ex. 38, 40, and 48.
Exercises

Identify each of the following subjects as to type, and write answers to same.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

Compose original subjects within the various types in both major and minor keys and write answers for same.

The Countersubject

The Countersubject is in effect a second subject that accompanies the subject and answer throughout the course of the fugue, or as long as the composer wishes it to do so. Generally, in the opening section of the fugue, it appears first against the answer (which normally is the second entry) and subsequently against the subject in the third entry. The following Bach quotation demonstrates in the simplest possible way how a countersubject operates.

\[ \text{Ex. 69 WTC II, Fugue No. 2 in C minor} \]

The subject, answer and countersubject are emphasized by the larger note-heads. The remaining material, in smaller notes, is free and not related to the subject and countersubject.

As is illustrated in the above example, a countersubject must be so constructed that it will function equally well both above and below the subject and answer. In order to achieve this result it is necessary to compose the countersubject so that it will operate in Double Counterpoint at either the 8ve or the 15th (i.e., the double 8ve) against the subject. In the present instance, the interval of inversion is Double Counterpoint at the 15th, thus

\[ \text{Ex. 70} \]

While Double Counterpoint at the 15th affords greater freedom of movement within the countersubject because of the larger interval of inver-
sion, it is sometimes expedient for textural purposes to construct the counter-subject within the narrower limitations of Double Counterpoint at the 8ve. By this means the subject and countersubject will be kept closer together. Such is the case in the following example where all of the intervals between the answer and countersubject are less than an 8ve.

Ex. 71

Bach: Three-Part Invention No. 8

The compositional process for writing the above countersubject would be a counterpoint invertible at the 8ve, thus:

Ex. 72

A somewhat unusual construction occurs in Fugue No. 23 in B major in WTC II, wherein the countersubject crosses the answer in the second entry. Later, when the countersubject is placed above the subject, the voice crossing does not come into play. The following two quotations from the fugue—measures 5 - 8 and meas. 19 - 22—show how this comes about.

Ex. 73

WTC II, Fugue No. 23 in B major

When a subject is so constructed that a tonal answer is required, the subject and answer are not alike with the result that a more complicated double counterpoint format is required. The subject and answer in Ex. 38

—Tonal Group II (a)—appear in the fugue, the operation of the countersubject in the second, third and fourth entries is as shown below.

Ex. 74

WTC I, Fugue No. 18 in G-sharp minor

The technicalities involved in writing countersubjects for all conditions involving tonal answers follow herewith.

Process For Composing A Countersubject

From the point of view of composing a countersubject, subjects fall into three general categories:

1. All of the subject is in the tonic key.
2. Most of the subject is in the tonic key.
3. Most of the subject is in the dominant key.

Every subject will fit into one of these categories.

1. WHEN ALL OF THE SUBJECT IS IN THE TONIC KEY

Subjects of this kind are those that require a Real answer. But, these in turn subdivide into two classes of contrapuntal construction:

(a) those without tied notes, and

Ex. 75

(b) those with tied notes.
Turning first to the subject and answer given in Ex. 75, wherein no tied notes occur, one can proceed in either Double Counterpoint at the 8ve or at the 15th. At every point in the structure the longer note, whether in the subject or in the countersubject, will serve as Cantus Firmus while the part in faster notes will act as Counterpoint. This is the same process shown in Ex. 70 and Ex. 72. The following is such a treatment of Ex. 75 in Double Counterpoint at the 15th.

When the above is transposed to its dominant key, in this case A minor, it automatically provides the countersubject for the answer.

In this case the two themes are never separated by an interval greater than a minor 7th, thereby facilitating the weaving of a tightly knit contrapuntal texture.

When a subject features tied or dotted notes—whether these be suspensions, retardations or simply ties—as in the following,

a specific order of steps in the composition of the countersubject may be suggested.

**Step 1** Treat all suspensions and retardations in the subject.
Step 2 Treat all suspension opportunities against the subject.

Step 3 Fill in the remainder of the countersubject and embellish the suspensions if so desired.

The discretion of the composer is of utmost importance here. This part of the procedure can either be overdone so that the countersubject becomes too complicated and “fussy,” or be made so sparse that the countersubject is ineffective. The degree of complexity must be in accordance with the composer’s artistic requirements. The composer should decide in advance whether he wants the countersubject to be

1. as complex as the subject,
2. more complex than the subject, or
3. less complex than the subject.

2. When most of the subject is in the tonic key

Subjects that fall into this category will generally be either Real with the Group IV(a) effect, as illustrated in Ex. 53, 54, 55, 58, and 59, or Group I as in Ex. 29, 30, and 31. There are rare cases in Group II wherein the modulation to the dominant key occurs so late in the subject that most of it is in the tonic key. This situation is illustrated in Ex. 40, 41, and 42.

A typical case would be the following subject which is in effect a Real (b) into which is inserted the Tonal Group IV(a) effect at the second note, being a V note approached by leap from the I.

To compose a countersubject that will operate successfully with both the subject and answer, proceed as follows:

Step 1 Set up a three-stave Double Counterpoint format with the subject on the top line, and on the bottom line the answer transposed to its subdominant key. The middle line is left blank for the countersubject. Now the subject and answer are so aligned that the tonic portions of the subject relate in Double Counterpoint at the 8ve while the single note in the dominant key relates to the corresponding tonic note in the answer in Double Counterpoint at the 9th.
Step 2 Fill in the countersubject on the middle line so that it will produce correct counterpoint against both subject and answer.

Ex. 86

Subject

Countersubject

Answer

transposed to its subdominant key

Step 3 Copy out the countersubject in Double Counterpoint against both subject and answer, the latter being retransposed back to its original key. The countersubject, too, must be transposed to match the key of the answer.

Ex. 87

Subject

Countersubject

Answer

transposed to its subdominant key

The same contrapuntal problem comes about in a somewhat different way when the subject begins with a dominant note as in Tonal Groups I and III. The following subject and answer can be classified as Tonal Group I(b) with a Tonal Group IV(a) effect inserted at the third note.

Ex. 88

Subject

Countersubject

Answer

transposed to its subdominant key

The Double Counterpoint format will be set up exactly as in Ex. 85, but in this case Double Counterpoint at the 9th will take place at both the first and third notes.

Ex. 89

Subject

Countersubject

Answer

transposed to its subdominant key

When the countersubject is filled in on the middle staff as was done in Ex. 86, it will now be expedient to begin the countersubject before the subject in order to make a suitable approach to the limited contrapuntal resources that are available in Double Counterpoint at the 9th.

Ex. 90

Subject

Countersubject

Answer

transposed to its subdominant key

It will be noted that although the countersubject is the same melodically for both subject and answer, its contrapuntal operation is different in the second part of the first measure where the dominant note occurs in the subject. Against the subject the countersubject produces a tie while against the answer it forms a suspension.
It can be noted parenthetically that when Bach constructed the countersubject for the subject illustrated in Ex. 69 and 70, he avoided any involvement with Double Counterpoint at the 9th by starting the countersubject after the initial dominant note of the subject.

When the countersubject developed above is shown against the subject and answer separately, with the answer in its original key, it will appear as in Ex. 87.

Ex. 91

3. WHEN MOST OF THE SUBJECT IS IN THE DOMINANT KEY

This kind of subject occurs mainly in Tonal Groups II and III, although it is quite possible to have a subject in Group I that meets this condition, as is illustrated in Ex. 32. The following illustration is a simply conceived subject within the Tonal Group III(b) framework, with the modulation to the dominant key coming between the repeated notes in the first measure (cf. Ex. 65).

Ex. 92

To compose a countersubject to operate with a subject and answer of this kind, proceed as follows:

Step 1  Set up a three-stave Double Counterpoint format with the subject on the bottom line, and on the top line the answer transposed to its dominant key. As in Ex. 85, the middle line is reserved for the countersubject. Here the subject and answer are so aligned that the dominant portions of the subject relate in Double Counterpoint at the 8ve while the note in the tonic key relates to the corresponding dominant note in the answer in Double Counterpoint at the 9th.

Ex. 93

Step 2  Fill in the countersubject on the middle line so that it will produce suitable counterpoint against both subject and answer.

Ex. 94

Step 3  Copy out the countersubject in Double Counterpoint against both subject and answer, the latter being retransposed back to its original key. As was the case in Ex. 87, the countersubject, too, must be transposed to match the key of the answer.
CHAPTER IV

The Stretto

By Stretto is meant the contrapuntal device wherein the subject or answer is played against itself canonically, or when the subject and answer are made to overlap. The following quotation from Fugue No. 2 in C minor (measures 23-27) from WTC II is an interesting and fairly complex example of stretto technique. The subject is the one in Ex. 31, demonstrating Tonal Group I (b).

Ex. 97

As demonstrated above, it is not necessary in a stretto formation that all entries of the subject or answer be stated with all intervals being used literally. The themes can be modified intervallically or used only partially. Often in a stretto modifications are made to achieve greater harmonic interest.

In measures 14-16 of the same fugue from which Ex. 97 is taken (WTC II, No. 2) there is an ingenious three-part stretto consisting of the subject in its original form as well as in augmentation and the answer in contrary motion with some intervallic alteration. The subject in augmentation is the middle voice and serves as cantus firmus. The passage appears as follows:

Ex. 98

The entire stretto technique can be crystallized into twelve specific types:

1. Literal
2. Contrary Motion
3. Retrograde
4. Augmentation
5. Diminution
6. Contrary Motion + Retrograde
7. Contrary Motion + Augmentation
8. Contrary Motion + Diminution
9. Retrograde + Augmentation
10. Retrograde + Diminution
11. Contrary Motion + Retrograde + Augmentation
12. Contrary Motion + Retrograde + Diminution

1. Literal Stretto

In the Fugue in F major, WTC I, No. 11, Bach makes interesting use of a straightforward stretto at the 8ve, both above and below. First, beginning at measure 25 it is presented between the two lower voices with a free part in the soprano.

Ex. 99

Next, beginning at measure 36 it appears as a three-part stretto in the key of D minor, the relative minor of the key of the fugue, beginning with the uppermost voice.

Ex. 100

Then, beginning at measure 46 another three-part stretto appears, this time in G minor, which is the relative minor of the subdominant key to that in which the fugue is written, and with the order of entries in the preceding stretto reversed; that is, starting with the lowest voice.

Ex. 101

In none of these three stretto structures does the answer appear. In each case the harmonic context is quite different, and in the third one (Ex. 101) the last measure of the subject is embellished into a continuous 16th-note figure.

A subject that will perform as shown in the three preceding examples can be structured as a canon at the 8ve, in Double Counterpoint at the 15th. This process will automatically produce one canon at the 8ve above, and another at the 8ve below. In Ex. 100 and 101 the first and third entries need not enter into the canonic formation as they never overlap. Were the subject longer, the canonic process would have to involve an invertible three-part canon instead of a two-part canon.

2. Stretto by Contrary Motion

A stretto in contrary motion can be constructed in two ways:
1. by exact intervocal imitation (i.e., a minor 2nd imitated by a minor 2nd, a major 2nd by a major 2nd, and so on), or
2. by inexact intervocal imitation (i.e., a minor 2nd imitated by a major 2nd, a major 2nd by a minor 2nd, and so on).

A remarkable feature of Bach's subjects is that they provide a great many stretto possibilities that he never utilized. Thus, in the remainder of the illustrations in this chapter some of Bach's subjects will be shown in stretto formations that do not actually appear in his fugues. Ex. 102 shows the subject of Ex. 97 in three contrary motion strettos with a free part added to demonstrate a possible harmonic context. At this point, however, it must be stressed that this example and the ones that follow that are not by Bach himself are in no way an attempt to suggest that this is something that Bach should have done. Obviously, he knew better than anyone else what his artistic requirements were and how to develop them. What is shown here is to be construed as nothing more than a technical demonstration of what can be done with a well-constructed subject if the composer wishes to do so. It would be silly to think that a composer of Bach's calibre was unaware of these stretto opportunities and how to use them. The only reasonable
assumption is that he simply saw no place for these strettos in his music. And, herein lies an important lesson for every composer: one must learn to become highly selective in his use of contrapuntal possibilities and reject those that are not artistically suitable to the needs of a particular composition.

The following example shows Bach's theme in three different strettos in contrary motion:

(a) at the 3rd above after two 8th-notes,
(b) at the 2nd above after two 8th-notes, and
(c) at the 7th below after four 8th-notes.

Thematically (b) and (c) are the same, but the relation to the original subject both in pitch and in time are different, thereby giving rise to different contextual harmonies. In all three strettos, however, the part that strettos is identical intervallically to the original subject.

Ex. 102

A curious four-part stretto with two parts moving in each direction is found in Fugue No. 3 of Klengel's Kanons und Fugen, Vol. I. The subject in D-flat major is Real (b) as follows:

Ex. 103

In measures 39-42 the following stretto appears. The 2nd, 3rd, and 4th entries are not complete. The two middle voices enter together in a Mirror Counterpoint. It is here transcribed into open score as the original piano format is visually very complicated due to the several part crossings.

Ex. 104

3. Retrograde Stretto

A retrograde stretto is one in which the subject or answer is played backwards against itself. This is what is commonly called either Crab Canon or Cancrizans Motion. It is encountered only very rarely. There are several reasons why a retrograde stretto may prove to be artistically impractical:

1. it cannot be recognized audibly,
2. many rhythms, especially dotted and tied notes, become disjointed and ineffective when heard backwards, and
3. a retrograde stretto is likely to set up awkward harmonic situations.

Ex. 105 is not from a fugue, but from a chorale harmonization, "Wer nur den lieben Gott lässt walten," No. 112 in the 371 Chorales by Bach. It will be observed that the first seven notes of the alto constitute a Real (d) subject (beginning and ending on the III note) in D major while the tenor in the corresponding notes is the answer in retrograde. From the purely theoretical standpoint this retrograde stretto is a Crab Canon constructed within the availabilities of Double Counterpoint at the 7th.

It can also be pointed out that while the subject and answer are strictly in D major the contextual harmony consists of a modulation from F-sharp minor to B minor via D major.

Ex. 105
The subject from Fugue No. 2 in WTC II that is shown in contrary motion strettos in Ex. 102 can likewise function in retrograde. Such a treatment with a possible contextual harmony is shown herewith. As is the case in Ex. 102, the following stretto treatment of this subject does not appear in the music of Bach.

Ex. 106

In the above example the crab canon that produces the retrograde strettos between soprano and tenor is cast in Double Counterpoint at the 17th; that is, Double Counterpoint at the 10th expanded by an octave. It is, of course, impossible to construct a crab canon within an even-numbered Double Counterpoint inversion.

4. Stretto by Augmentation

A stretto by augmentation by Bach himself is shown in Ex. 98, between soprano and tenor.

It is customary to think of augmentation in doubled note values; that is, in the proportion of 1 to 2. But, there is no reason why other proportions of augmentation should not be employed. Ex. 107 shows a strettos in triple augmentation; namely, in the proportion of 1 to 3.

Ex. 107

Neither the strettos nor the subject shown above are to be found in any existing fugue. It was contrived simply to illustrate the contrapuntal possibility. A strettos in triple augmentation is impractical unless the subject is designed especially for it. Only a subject in triple rhythm will lend itself to triple augmentation.

5. Stretto by Diminution

Structurally and mechanically there is no difference between strettos by augmentation and strettos by diminution. If the original subject is in shorter notes, the strettos is by augmentation, and if the original subject is in longer notes the strettos is by diminution. But, canonically the process of construction is the same. Ex. 109 quotes measures 30-32 of Fugue No. 9 in WTC II with the subject in its original form in the alto and the answer in diminution in the bass. This subject is given also in Ex. 5.

Ex. 109

6. Stretto in Contrary Motion and Retrograde

This kind of strettos entails a crab canon in contrary motion. The versatile subject from Fugue No. 2 in C minor in WTC II makes such a strettos available, as shown below. Again, this strettos does not appear in Bach's music. It is merely another in the series of adaptations made for demonstration purposes.

Ex. 110
7. Stretto in Contrary Motion and Augmentation

The ingeniously contrived subject of Fugue No. 2 in WTC II again makes possible the demonstration of a stretto that does not appear in the fugue; namely, a stretto in contrary motion and augmentation.

Ex. 111

![Subject in Contrary Motion and Augmentation](image)

8. Stretto in Contrary Motion and Diminution

A stretto of this type is worked into a massive stretto involving all four parts in measures 9-12 of Fugue No. 9 in E major in WTC II. It begins on the III note in contrast to the other four entries which consist of two subjects and two answers that begin on the I and V notes in the usual way. Also, the entry in contrary motion and diminution is slightly disguised by the addition of a passing-note as indicated in the following example.

Ex. 112

WTC II, Fugue No. 9 in E major

![Subject in Contrary Motion and Diminution](image)

9. Stretto in Retrograde and Augmentation

Once again the subject from the Fugue in C minor, No. 2 in WTC II, makes possible an example of a stretto in retrograde and augmentation. The stretto is in the two outside voices with two free parts between them. This stretto was contrived by means of Bach's subject and does not appear in the fugue.

Ex. 113

![Subject in Retrograde and Augmentation](image)

10. Stretto in Retrograde and Diminution

This stretto is made possible by the subject from the Fugue in B-flat minor, No. 22 in WTC I. As in the case of several of the preceding illustrations, this stretto does not appear in Bach's music.

Ex. 114

![Subject in Retrograde and Diminution](image)

11. Stretto in Contrary Motion, Retrograde and Augmentation

The remarkable canonic qualities of the strangely chromatic subject of the Fugue in F minor, No. 12 in WTC I, makes possible a stretto of this type. Due to the chromatic structure of the subject the resulting harmonies are most unusual. This stretto does not appear in Bach's music.

Ex. 115

![Subject in Contrary Motion, Retrograde and Augmentation](image)

12. Stretto in Contrary Motion, Retrograde and Diminution

The subject used in the preceding stretto also makes possible the present stretto in Contrary Motion, Retrograde and Diminution, which, as in the case of Ex. 115, does not appear in Bach's music. The harmonies resulting from this stretto are even more unexpected than those in the preceding example.

Ex. 116

![Subject in Contrary Motion, Retrograde and Diminution](image)
Solving the Stretto Problem

The stretto problem confronts the student in two ways:
1. constructing strettos to a given subject, as in a fugue examination; and
2. constructing an original subject to yield pre-planned stretto combinations.

In order to demonstrate the first situation a subject attributed to Matthe-son and used by Bach in his C major Fugue for unaccompanied violin will be treated according to a definite and highly organized system. The subject, Tonal Group I (b), and its correct answer follow.

Ex. 117

Subject

Answer

Procedure

First, fit the beginning of the subject against the end of the subject in as many ways as possible. For instance, by means of a 7th chord it is possible to achieve a stretto at the octave below as follows:

Ex. 118

A stretto at the 8ve, such as the above is probably the one most likely to be encountered in a traditional fugue. The harmony shown above is likewise the one that is most likely to be used, but this should not deter the student from trying more venturesome harmonizations. Strettos at any of the following intervals are, of course, possible and probably more interesting than the one at the 8ve in Ex. 118.

At the 7th below:

Ex. 119

In the above example the entrance on “A” is the subject transposed literally to the key of D major. Quite different harmonic results would occur if flats were placed by the “A” and “B” with the F-natural, thus:

Ex. 120

At the 6th below:

Ex. 121

At the 5th below:

Ex. 122

In this case the entry of the subject at a 5th below brings about the subdominant key, both harmonically and thematically.

At the 4th below:

Ex. 123
This brings about a unison at the last note which could weaken the harmonic and contrapuntal texture at that point.

When the stretto is at a larger interval it may prove expedient not to place it in two adjacent voices. The following shows the same subject in a stretto at the 9th below in alto and bass, with free parts in soprano and tenor.

Ex. 124

Next, in the same manner try a stretto at every interval above. Ex. 125 shows a stretto at the 5th above, which places it in the dominant key so far as the theme itself is concerned. However, the contextual harmony may turn out to be in some other key, in the present instance in D minor. This does not mean that the harmony will remain in D minor throughout the stretto.

Ex. 125

After all strettos at this entrance point have been explored, the next step is to move back a half measure, or a full measure as desired and repeat the process. The following examples show a few of the stretto possibilities a half measure earlier.

At the 4th above:

Ex. 126

The stretto shown above can invert in Double Counterpoint at the 9th. In this form it could be placed in alto and tenor thus:

Ex. 127

The above strettos are shown with a possible contextual harmony in the section where the themes overlap. Whether a stretto is artistically practical depends in large measure upon the harmony it produces. In many cases this harmony may not fit with that of the rest of the fugue, and when such a situation comes about the stretto in question may have to be rejected, although it is entirely possible theoretically.

At this same point a Retrograde stretto is possible.

Ex. 128

The above retrograde stretto can be inverted in Double Counterpoint at the 8ve, 9th and 11th, with different harmonies being produced at each Double Counterpoint inversion.

Ex. 129

(a) Double Counterpoint at the 8ve

(b) Double Counterpoint at the 9th

(c) Double Counterpoint at the 11th

Moving back another half measure a Stretto in Contrary Motion comes into play. This stretto is invertible in Double Counterpoint at the 6th. Both the original stretto and its inversion are shown below. In the inversion at Double Counterpoint at the 6th the flat is inserted to avoid the crude outlining of the Augmented 4th melodically. It goes without saying that in any stretto formation accidentals may be added freely in order to

1. make the harmony more interesting,
2. avoid awkward contrapuntal or harmonic situations, or
3. carry out a pre-determined modulation plan.
Ex. 130

Subject (a)

(b) Double Counterpoint at the 6th

Beginning at the first full measure, two excellent strettos of the answer against the subject are available: at the first part of the measure the answer can enter below the subject, and at the third quarter in the measure it can enter above the subject, thus:

Ex. 131

(a)

(b)

The strettos in Ex. 118-131 are given without any consideration of quality or musical effectiveness. They are presented purely on a technical basis to provide a workable method by which a student can assess quickly just what the stretto possibilities of a given subject are. Once all of the available stretto combinations of a subject are recognized, it is up to the composer to exploit only those that meet his artistic needs.

Composing a Subject with Pre-established Stretto Requirements

All of the preceding strettos have been developed out of a given subject. The stretto problem becomes somewhat different and under a different kind of control when a composer constructs a subject to yield specific preplanned stretto formations. Since every stretto is a canon, the reader is referred to THE TECHNIQUE OF CANON by Hugo Norden (Branden Press, Boston), wherein every kind of canon is discussed together with the exact process for composing it.

To compose a subject to meet specific stretto requirements proceed as follows:

Step 1 Formulate the problem. For example:

Compose a subject of type Real (b) in the key of D major, 2/4 rhythm, 3 measures long (i.e., ending on the 4th measure), to yield a stretto at the 8ve above after 1 measure.

Step 2 Set up the canon problem so that the first and last notes will conform to the terminal specifications stated in Step 1.

Ex. 132

Step 3 Complete the canon. The following illustration shows three solutions to the above problem.

Ex. 133

(a)

(b)

(c)

When two different strettos at the same time distance are required, the problem will entail a canon in the Double Counterpoint resulting from the sum of the intervals of the two strettos. For example,

Compose a subject of Tonal Group II(a) in the key of F major, 4 measures long (ending on the fifth measure) in 3/4 rhythm to yield strettos at the 5th above and at the 6th below after 2 measures.

Step 2 would call for a canon problem in Double Counterpoint at the 10th as shown below.
Step 3 could be carried out rather simply as follows.

A more complex problem might involve a canon in three or more parts. For example, a problem could be stated thus:

Compose a subject of type Real (b) in the key of C major, three measures long (i.e., ending at the 4th measure) to yield strettos as follows:

(1) at the 5th above after 1 measure,
(2) at the 8ve below after 1 measure, and
(3) at the 4th below after 2 measures.

These are all contained in a three-part canon constructed within Double Counterpoint at the 12th, (1) and (2) being the calculated two-part canons and (3) being the uncalculated two-part canon within the three-part canon structure.

Step 2 could appear thus:

The strettos specified in the problem and worked out by means of whatever canon structure is appropriate will usually not be the only strettos that such a subject will yield. If a subject composed in this way is then examined as shown in Ex. 118-131, it will undoubtedly be found that a great many other strettos of the same subject are available.

Other kinds of stretto involving contrary motion, retrograde, augmentation, and diminution can be pre-planned in the same way by canons in contrary motion, retrograde, etc. Any composer with an adequate technique in canon writing should not experience any difficulty in composing subjects that will stretto in any way he requires.
The terms codetta and episode both refer to connecting passages that are generally composed of free material that may be either similar to or different from motives found in the subject or countersubject. The codetta is used to lead to a new entrance of the subject or answer in the exposition, while the episode is designed to link entries of the subject or answer after the exposition is completed.

The first codetta appears at the end of the initial statement of the subject when there is a time lag between the end of the subject and the beginning of the answer. In such a case it is usually the function of the first codetta to form an unbroken line from the end of the subject to the beginning of the countersubject which is played against the answer when it comes in as the second entry of the fugue. The three quotations that follow show how this comes about.

Ex. 138
(a) WTC I, Fugue No. 7, in E-flat major

(b) WTC II, Fugue No. 23, in B major

(c) Art of the Fugue, Contrapunctus 8

When the end of the subject and the beginning of the answer overlap so that no time lag exists, as in Ex. 139, there is obviously no place for a codetta at this point in the fugue.

Ex. 139
WTC II, Fugue No. 2, in C minor

The codetta between the second and third entries is usually longer and more substantive than the one between the first and second entries. In Ex. 140 no codetta occurs between the first and second entries while a two-measure codetta is employed to link the answer to the second appearance of the subject. The lower part in the codetta reappears later in the fugue and becomes a unifying element in the form of the composition as a whole.
A four-voice fugue has a place for a codetta between the third and fourth entries. Since there have been three entries before such a codetta, it will have to be in three-part counterpoint, and may be of considerable complexity texturally. Ex. 141 shows such a codetta, and it is interesting to note that there is no codetta before this one as no time lag was built in between the first and second and between the second and third entries. In the present instance, the codetta develops imitatively thematic fragments from the countersubject.

The episode differs from the codetta chiefly in its location in the fugue, which is after the exposition; that is, after all the voices have come in with the subject and answer for the first time. After the exposition in an intelligently planned fugue, the entries of the subject and answer, whether singly or in stretto, will occur according to a prearranged plan. For instance, in Bach's Fugue in C minor, WTC I, No. 2, five entries occur at the second eighth-note in odd-numbered measures. This plan brings about a subject in C minor in the bass at measure 7 and a subject in the relative major key of E-flat at measure 11, thereby leaving measures 9 and 10 open for an episode.
The connecting episode as Bach wrote it links these two entries as follows.

Ex. 143

A well-constructed episode can be a miniature composition in itself, a demonstration of the “wheels within wheels” principle. A careful examination of the above example will show that structurally it consists of a canon at the 5th in the two upper parts accompanied by a running bass line. Still closer examination reveals that the canon is made up of a sequential treatment of the head of the subject while the running bass part is correspondingly sequential and is derived from the beginning of countersubject I. The component elements of the episode are identified in the following diagram.

Ex. 144

The next scheduled entry is an answer that comes in at the second eighth-note in measure 15 in the alto part accompanied by countersubject I in the uppermost part. Thus it is necessary to insert an episode from the beginning of measure 13, the last note in Ex. 143 to the beginning of measure 15. This episode calls for a modulation from E-flat major at the beginning of measure 13 to C minor at the beginning of measure 15. It embodies the following features:

1. These are the only two measures so far that have no rests.
2. The running passage derived from the opening motive of countersubject I is placed in the uppermost part with the direction of the runs reversed, ascending instead of descending.
3. The two lower voices move in parallel 3rds that are based on the second motive of countersubject I. No fragments of the subject itself appear in this episode.
4. This episode ends on the highest note in the entire fugue, the high ‘C’, which appears nowhere else in the composition.

The episode proceeds as follows:

Ex. 145

The third episode serves quite a different purpose than the first two, in that it leads to the sixth entry which is the first time that the subject enters in an even-numbered measure. This entry of the subject at the second eighth-note of measure 20 is extremely important in that it defines the proportion scheme of 19:12 within the 31-measure time span of the fugue as a whole. The problem can be seen in the following form, with the three measures between the end of the answer in the fifth entry and the beginning of the subject in the sixth entry.

Ex. 146

This episode is built around the opening motive of the subject in combination with the theme in the codetta shown in Ex. 140. This codetta theme appears only in the episodic sections. Compare this episode with the codetta in Ex. 140.
The fourth episode has a still quite different task to perform, namely, to lead to an entry of the subject in the bass beginning at the 6th eighth-note in measure 26. This is the first and only entrance that occurs at the 6th eighth-note in an even numbered measure. This is the longest episode in the entire fugue, four and one-half measures, beginning in measure 22 and extending to the middle of measure 26. The problem can be visualized thus:

Ex. 148

This episode is constructed in much the same way that the first episode is written (cf. Ex. 144), with a canon at the 5th in the two upper parts above the running passage in the bass. Thus, the first and fourth episodes by being almost identical structurally serve to unify the composition as a whole. The episode is diagrammed in detail in Ex. 149, but compare it carefully with episode 1 as shown in Ex. 144.

Ex. 149

The rhythmic difference between the upper voice of the canon in the second half of measure 23 and the lower voice in the first half of measure 24 serves a highly practical purpose. By means of this rhythmic alteration measure 24 has no rests, as is the case in measures 13 and 14. These three measures are the only ones in the entire fugue that have no rests.

Measures 13 and 24 mark the beginning and end of a 12-measure section that is of formal significance within the 31-measure form, which is divided in at least three ways into the 12:19 proportion. The operation of this compound proportion scheme is as follows:

1. 12 measures before measure 13 where there are no rests, and 19 measures from there on.
2. 19 measures before the first entry in an even-numbered measure in measure 20, and 12 measures from there on.
3. 12 measures within the portion that begins and ends with measures without rests (measures 13-24), and 19 measures in the rest of the fugue.

Only one more short episode remains to be examined; namely, from the middle of measure 28 to the middle of measure 29, where the last entry of the subject and the tonic pedal-point in the bass begin.

Ex. 150

In the fugue presently under examination the following components are seen in Ex. 140-150:

8 entries—5 subjects in C minor, 1 in E-flat major, and 2 answers,
1 codetta,
5 episodes.

When these are "assembled" to utilize the 31-measure form plan, the complete fugue appears as in Ex. 151.
The episodes examined in Ex. 142-150 determine the form plan of a three-part fugue, WTC I, No. 2, in C minor. The thematic material therein consists of a subject and two countersubjects that operate in triple counterpoint. However, countersubject II is treated so freely that at times it can almost be considered as a free part.

In a four-part fugue the problem is not significantly different. Such a fugue is the Fugue in G minor, WTC I, No. 16.

The subject is Tonal Group I(b), beginning on the dominant note and ending on the mediant note. Thus, the head of the subject and that of the answer are not alike at the opening interval, so that the effect of the entries of the subject and those of the answer are quite different.

The countersubject is derived from the closing motive of the subject in contrary motion, thereby giving unity and continuity of the rhythmic and melodic flow of the entire composition. The three themes—subject, answer and countersubject—can be isolated as follows.
The complete fugue appears as in Ex. 153.

Ex. 153 (continued)
The complete fugue appears as in Ex. 153.

Ex. 153 (continued)
The form plan of Fugue No. 16 in WTC I as determined by the entries of the subject, answer and the episodes can be tabulated as follows:

- **Measure 1, 2nd 8th-note:** Subject in alto
- **Measure 2, 6th 8th-note:** Answer in soprano
- **Measure 4:** CODETTA in soprano and alto
- **Measure 5, 2nd 8th-note:** Subject in bass
- **Measure 6, 6th 8th-note:** Answer in tenor
- **Measures 8-11:** EPISODE 1
- **Measure 12, 2nd 8th-note:** Subject in B-flat major in alto
- **Measure 13, 6th 8th-note:** Answer in B-flat major in bass
- **Measure 15, 2nd 8th-note:** Subject in F major in soprano
- **Measure 17, 2nd 8th-note:** Subject in B-flat major in bass
- **Measure 19:** EPISODE 2
- **Measure 20, 2nd 8th-note:** Subject in C minor in bass
- **Measure 21, 6th 8th-note:** Subject in C minor in soprano
- **Measure 23, 2nd 8th-note:** Answer in C minor in alto
- **Measures 24-28:** EPISODE 3
- **Measure 28, 2nd 8th-note:** Subject in G minor in soprano
- **Measure 29, 2nd 8th-note:** Subject in G minor in bass
- **Measures 30-31:** EPISODE 4
- **Measure 31, 6th 8th-note:** Subject in G minor in alto
- **Measure 33, 2nd 8th-note:** Subject in G minor in tenor

From the above tabulation it will be seen that the fugue embraces 17 entries of the subject or answer (half as many as there are measures in the composition), two strettos, one codetta, and four episodes. Slightly over one third of the time span of the fugue is given over to episodic material.

There is not one full measure without a rest. And only a very small portion of the composition contains four-part harmony. In most instances students tend to overharmonize with the result that they find themselves struggling with contrapuntal textures that are far too thick and excessively cumbersome.

Except for the first and the penultimate measures, there is not a measure in the entire fugue that does not include the thematic material of the end of the subject and the beginning of the countersubject. The episodes, however, are distinguished by new and unrelated thematic material, but this is played against that of the subject and countersubject.

This economy of material and texture is very important in achieving unity and continuity and merits careful study.

No pedal-points are included. But, if the bass ‘D’ were held throughout measure 28 and into measure 29 it would make a dominant pedal; so this could be considered a pedal-point by implication.

The fugue with all of its component parts are diagrammed in Ex. 154.
It may be helpful to crystallize a process for the writing of an episode. As a practical example, let us suppose that a degree candidate is writing an examination fugue on the following subject and countersubject. The problem is a relatively simple one as the subject is a Real (b) in that it begins on the tonic note and ends on the mediant. The countersubject has been constructed in Double Counterpoint at the 8ve so that no inversion problems will come about.

As the fugue has been plotted out, there is a section in which an entry of the subject appears in the uppermost voice, the countersubject in the bass and a free part in the middle. After a three-measure episode the plan calls for a subject in the bass, the countersubject in the uppermost voice and a free part in the middle. The problem is set up thus:

It is imperative that the harmonies at the two ends of the episode are clearly defined before the actual writing of the passage is undertaken. It is of utmost importance that the composer has an exact picture in his mind of what he is going from and what he is going to. Otherwise there can be no intelligent and precise sense of direction.

The next step is to decide on a contrapuntal or harmonic device to direct the contrapuntal texture from the beginning to the end of the episode, and insert this in the space allotted for the episode. In the present instance it has
has been decided to use a canon in contrary motion which is derived from the
opening motive of the countersubject. This choice is purely arbitrary. It might
have been a series of harmonies to be activated contrapuntally or a line to
serve as cantus firmus or any other type of imitative or even a non-imitative
device. However, the choice has been made for the canon in contrary
motion and it is inserted as follows.

Ex. 157

The final step in the composing of an episode is to connect it artistically
at both ends so that there are no "seams" or "rough edges" to disrupt the
smooth flow of the composition as a whole. This part of the process may
well be the most difficult and demanding artistically. Example 158
represents an attempt to complete the episode developed in Ex. 156-157. The
connecting material that lies in the canon in contrary motion and its accom-
panying free bass part to the entries that precede and follow the episode is
indicated by brackets. If such joinings are so smooth and so elegant that the
listener remains unaware of them, the composer is to be complimented.

Ex. 158

CHAPTER VI
The Pedal-point and Cadenza

Academic exercises and examinations in fugue generally require two
pedal-points, one on the dominant note to be followed by another on the
tonic note. The main purpose of these two pedal-points is to reestablish the
tonality of the composition in the event that it has become clouded by the
numerous modulations and digressions along the way within the middle
entries of the subject and answer and in the connecting episodes. However, it
is safe to say that most of the fugues in the literature do not contain extended
pedal-points. Thus, the pedal-point, as a fugal device, can be thought of as
largely, though certainly not entirely, an academic feature.

Two unusually interesting pedal-points by Bach occur in WTC II, Fugue
No. 1, in C major. These are quoted in full in Ex. 159 and 160. The measures
are numbered so that it can be readily seen where in the 83-measure fugue
these occur. The first one consists of a tonic pedal-point in the uppermost
voice that dovetails with a dominant pedal-point in the bass. Against these
two interlocking pedal-points the subject and answer are played respectively.
The free running passages in 16th-notes are derived from the second half of
the subject. As noted by the measure numbers, this pedal-point format
begins at measure 47, five measures after the middle of the 83-measure form.
It is extremely rare to find a pedal-point in the first half of a fugue.

Ex. 159

The second of the two pedal-points in Fugue No. 1 of WTC II is
considerably longer and employs only the tonic pedal-point, and begins at
measure 68, well within the last quarter of the 83-measure form. Unlike
the earlier pedal-point shown above, this one employs only the head of the
subject which is made to operate in double counterpoint against a 16th-note
passage derived from the second half of the subject. These two themes are
identified by a single line and a double line respectively. It will be noted
that when these two themes appear in measures 72-75, they are identical
to the presentation in measures 68-71 except that they have been inverted
in Double Counterpoint at the 8ve, all of which takes place against the tonic
pedal-point which appears first between the two themes and afterwards
above them. When the pedal-point is taken into consideration contrapuntally,
this passage can be thought of as being in triple counterpoint.
In Ex. 100 is shown a stretto above a dominant pedal-point from the Fugue in F major, No. 11 in WTC I. However, since the fugue is in F major, the pedal-point is on the dominant of D minor, the relative minor key to F major. The location of this pedal-point is important. It goes from measure 36 into measure 40, thereby spanning the middle of the 72-measure form within which the fugue is cast. This is the only pedal-point in the F major fugue. It could, of course, be thought of as coming on the mediant note of F major in spite of the fact that it operates harmonically in D minor.

A somewhat more ambitious cadenza occupies the last four measures of Bach's 96-measure Fugue in D minor for organ. It is actually in three sections over an implied dominant pedal-point. What is of particular interest in this case is the diversity of texture that makes the three sections so completely different from each other within so short a time span, and yet so unified in musical effect when performed.

A version of the same passage for unaccompanied violin closes the Fugue in G minor from the Sonata No. 1. The violin fugue is much like the organ fugue, but is two measures shorter, being only 94 measures.
CHAPTER VII
The Exposition

The exposition of a fugue is quite simply the opening section of the composition wherein each voice is brought in singly, alternating between subject and answer.

The exposition of Fugue No. 18 in WTC I, in G-sharp minor, is given in Ex. 74. The fugue is in four parts; it has a two-measure subject; the entries are not separated by codettas, so the entire exposition is only 8 measures long. The entrance scheme is as follows:

\begin{align*}
\text{Soprano:} & \quad \text{Subject} & \quad \text{Countersubject} \\
\text{Alto:} & \quad \text{Answer} & \quad \text{Countersubject} & \quad \text{Free part} \\
\text{Tenor:} & \quad \text{Subject} & \quad \text{Countersubject} & \quad \text{Free part} & \quad \text{Free part} \\
\text{Bass:} & \quad \text{Answer} & \quad \text{Subject} \\
\end{align*}

The final entry of the exposition is reserved for the bass. The free parts are shown in small notes to contrast with the subject, answer and countersubject.

The exposition of a three-part fugue, No. 2 in WTC I, is given in Ex. 140, this having a two-measure codetta between the second and third entries. The entrance scheme is carried out thus:

\begin{align*}
\text{Soprano:} & \quad \text{Subject} & \quad \text{Countersubject} & \quad \text{Free part} & \quad \text{Codetta} \\
\text{Alto:} & \quad \text{Answer} & \quad \text{Countersubject I} & \quad \text{Countersubject II} \\
\text{Bass:} & \quad \text{Subject} & \quad \text{Subject} \\
\end{align*}

In this instance, Countersubject II replaces the free part thereby making unnecessary the invention of any additional thematic material.

A four-voice exposition with a codetta between the third and fourth entries is given in Ex. 141, the fugue under examination being No. 12, in F minor, in WTC I. The entrance plan follows:

\begin{align*}
\text{Soprano:} & \quad \text{Subject} \\
\text{Alto:} & \quad \text{Answer} & \quad \text{Countersubject} & \quad \text{Free part} & \quad \text{Codetta} & \quad \text{Free part} \\
\text{Tenor:} & \quad \text{Subject} & \quad \text{Countersubject} & \quad \text{Free part} & \quad \text{Codetta} & \quad \text{Free part} \\
\text{Bass:} & \quad \text{Subject} & \quad \text{Countersubject} \\
\end{align*}

This exposition differs from that in Ex. 74 in three ways:

1. the order of entries is so planned that the soprano takes the final entry,
2. the final entry is a subject instead of the usual answer, so that the exposition consists of three subjects and one answer instead of the usual two subjects and two answers, and
3. the third and fourth entries are separated by a three-measure codetta. In Ex. 74 there is no codetta.

The entrance plan of the fugue examined in Ex. 154 requires special consideration. This is WTC I, Fugue No. 16 in G minor. It will be noted that
the final entry of the exposition, beginning in measure 6, comes in the tenor. Generally in academic exercises the student is discouraged from placing an exposition entry between two voices that are already in motion lest the entrance lose its impact by being covered over by the surrounding counterpoint. However, it can be pointed out that at the time of the fourth entry in the tenor, the soprano and alto are resting so that there actually is no surrounding counterpoint to conceal the answer. In fact, the texture is so thinned out that at the beginning of measure 7 only the answer is being heard. The complete entrance plan follows:

Soprano: Answer ————-Countersubject Free part
Alto: Subject Countersubject ————-Free part Rest
Tenor: Answer
Bass: Subject Countersubject

In a three-voice fugue there are six possible orders of entry, two of which present the added problem of bringing in the final entry between two voices already in progress. A four-voice fugue makes available 24 orders in which the exposition entries can be brought in, but 16 of these necessitate the bringing in of entries between voices already in progress.

For the student about to compose a fugue or to write an examination a systematic order of steps is available. To illustrate the process, subject No. 3 of those given at the close of Chapter II will be used.

Step 1 Write the correct answer and compose a suitable countersubject.

Step 2 Decide on an entrance arrangement, and write in the subject, answer and countersubject where desired. (The exposition to be developed is for a three-voice fugue.)

When the free part momentarily creates a chord of the 7th (whether complete or incomplete) as at *), or causes one of the other voices to become
the 7th of a chord as at **), the counterpoint should ordinarily be so controlled that the note that is the 7th is correctly resolved.

A special situation comes about in the setting up of an exposition when the fugue is either for voices or for instruments of a limited range. For example, a composer about to write a fugue on the following subject

Ex. 168

for mixed chorus could proceed as follows:

**Step 1** Write the answer and compose a suitable countersubject, being certain that the double counterpoint is satisfactory as this is most important in choral composition.

**Step 2** Tabulate the ranges of the subject, answer, and countersubject to both the subject and answer in both the treble and bass clefs in order to ascertain which themes are suitable for the various voices. This is done in Ex. 169 under Ranges.

Ex. 169

From the above tabulation it will be noted that the countersubject to the subject does not fall within a convenient range for the bass. Thus, an exposition should be devised in which this countersubject is never assigned to the bass. An entrance scheme for an exposition that avoids such an awkward vocal situation for the bass is suggested below.

Ex. 170

A rough and ready rule for writing vocal fugues is to place the countersubject to the answer in approximately the same range as the subject, which will automatically put the countersubject to the subject in about the same range as the answer. In this way the vocal continuity of each voice part will be assured.
CHAPTER VIII

Harmony

Fugal harmony must be considered on three levels:

1. The key scheme of the composition as a whole.
2. The underlying chordal structure.
3. The vertical relationship between the several voices.

The key scheme of the fugue as a whole is generally defined by a few definitive cadences. For instance, in WTC I, Fugue No. 2 in C minor, as given in full in Ex. 151, the following overall plan comes into focus:

Beginning in C minor.

measure 11: a powerful cadence in E-flat major followed by a statement of the subject accompanied by the two countersubjects in E-flat major.

measure 17: an equally powerful cadence in G minor followed by an episode.

measure 23: a cadence in B-flat major preceded by a less conspicuous cadence in C minor.

measure 27: a deceptive cadence in C minor, bringing in for the first time the A-flat chord at the beginning of a measure.

measure 29: final cadence in C minor.

Thus, the broad outline of the fugue as a whole could be seen as:— C minor—E-flat major—G minor—B-flat major—C minor— (A-flat major as VI in C minor)—C minor.

Some such comprehensive plan is necessary to give the entire composition unity and a sense of direction. Otherwise, there is the danger of a fugue "losing its way" through a number of unplanned middle entries and episodes that come and go without purpose or destination.

The underlying chordal structure of a florid fugal passage is closely akin to chorale harmonization as Bach practised this great art. Herein lies an unlimited field for analytical study in the music of Bach and all other composers. One brief example will suffice to illustrate how to proceed. Picked entirely at random is a short excerpt from Contrapunctus 7 of the Art of the Fugue. The passage chosen begins at measure 20 and extends into the beginning of measure 23.

Ex. 171

Art of the Fugue, Contrapunctus 7

77
In order to see the underlying chordal structure it is necessary to remove all embellishing materials. These are as follows:

- passing-notes,
- auxiliary-notes,
- suspensions,
- appoggiaturas,
- echappes,
- rhythmic embellishments, such as rests, note repetitions, etc.,
- other notes of the harmony that do not play a part in the progression process.

Shorn of all of the above embellishments the passage in Ex. 171 appears thus:

Since this underlying harmony contains no E-natural or E-flat it can be considered as being either in the key of F major or in the key of B-flat major. The passage is absolutely neutral insofar as the tonality is concerned. Thus, the feeling of a modulation from B-flat major to F major occurs only in the passing-notes and auxiliary-notes, and not in the harmonies as such. In contrapuntal harmony this is an extremely valuable technique, that is used extensively in the music of Bach. However, it should not be assumed that the chordal background of fugal harmony must be so neutral tonally. Extensive research will put this aspect of what has proved to be successful harmony in perspective.

Simplifying the harmony even further one discovers that there is a contrapuntal background behind the chordal structures. The following diagram shows how the chordal structure is the effect of two pairs of lines moving in parallel 10ths operating simultaneously. These can be identified by the dotted lines in contrast to the dot-and-dash lines. In the second measure the “F” in the soprano that becomes the 7th of the chord in the last quarter of the measure is not resolved in the harmony itself, but by the “E-flat” passing-note that has been deleted (cf. Ex. 171).

It would be difficult to say whether or not the above passage is typical of the chordal structure behind florid fugal harmony. Bach’s technique in this respect is as varied as it is in the chorale harmonizations. But, it can be stated as a principle that the more static the chordal progressions may be, the more is the opportunity for the embellishing discords to function freely and effectively.

The vertical relationship of the several voices can best be seen through the so-called “Two-Voice Principle”:

Every multi-voiced structure consists of a triangular number of two-part combinations. If all of these constituent two-part combinations are correct, it follows that the structure as a whole is correct. In a four-voice texture the two-part combinations are as follows:

1. Bass — Tenor
2. Bass — Alto
3. Bass — Soprano
4. Tenor — Alto
5. Tenor — Soprano
6. Alto — Soprano

A reliable, but probably a bit too conservative, method for determining the correctness of a passage can be formulated from the following table.
The two-part combinations:

1. Bass—Tenor
   - Must not contain a 4th except
     1. as a correctly treated dissonance
     2. in a correctly treated six-four formation
     3. in a correctly resolved inversion of a 7th or 9th chord
   - Must not contain
     1. The doubling of any note that must be resolved in only one direction, such as
        (a) the leading-tone,
        (b) a suspension,
        (c) the 7th or 9th of a chord,
        (d) chromatically treated notes that require resolution such as those in an Augmented 6th Chord.

2. Bass—Alto
   - Must not contain
     1. The doubling of any note that must be resolved in only one direction, such as
        (a) the leading-tone,
        (b) a suspension,
        (c) the 7th or 9th of a chord,
        (d) chromatically treated notes that require resolution such as those in an Augmented 6th Chord.

3. Bass—Soprano
   - Must not contain
     1. The doubling of any note that must be resolved in only one direction, such as
        (a) the leading-tone,
        (b) a suspension,
        (c) the 7th or 9th of a chord,
        (d) chromatically treated notes that require resolution such as those in an Augmented 6th Chord.

4. Tenor—Alto*)
   - Must not contain
     1. The doubling of any note that must be resolved in only one direction, such as
        (a) the leading-tone,
        (b) a suspension,
        (c) the 7th or 9th of a chord,
        (d) chromatically treated notes that require resolution such as those in an Augmented 6th Chord.

5. Tenor—Soprano
   - Must not contain
     1. The doubling of any note that must be resolved in only one direction, such as
        (a) the leading-tone,
        (b) a suspension,
        (c) the 7th or 9th of a chord,
        (d) chromatically treated notes that require resolution such as those in an Augmented 6th Chord.

6. Alto—Soprano*)

*) Should not be separated by an interval greater than an 8ve.

**) The 2-1 suspension is to be found with some frequency between Tenor and Bass.

As suggested previously, the above table may be considered by some as being too conservative for a sophisticated fugal technique but it provides two things:

1. A norm of correctness, especially for the preparation of work to be submitted for academic evaluation, and
2. A norm of correctness by which one can put in perspective the liberties that all venturesome composers take.

It was mentioned previously that every multi-voiced structure embodies a "triangular number" of two-part combinations. These are as follows:

<table>
<thead>
<tr>
<th>Multi-voiced Structure</th>
<th>Two-Part Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-part structure</td>
<td>1</td>
</tr>
<tr>
<td>3-part structure</td>
<td>3</td>
</tr>
<tr>
<td>4-part structure</td>
<td>6</td>
</tr>
<tr>
<td>5-part structure</td>
<td>10</td>
</tr>
<tr>
<td>6-part structure</td>
<td>15</td>
</tr>
<tr>
<td>7-part structure</td>
<td>21</td>
</tr>
<tr>
<td>8-part structure</td>
<td>28</td>
</tr>
</tbody>
</table>

These numbers, that is, the two-part combinations within a multi-voiced structure, can be seen in the following triangle. The first row is, of course, 1; the sum of the first two rows, 3; the sum of the first three rows, 6; and so on.

2-part structure = 1 two-part combination
3-part structure = 3 two-part combinations
4-part structure = 6 two-part combinations
5-part structure = 10 two-part combinations
6-part structure = 15 two-part combinations
7-part structure = 21 two-part combinations
8-part structure = 28 two-part combinations